

## Tilburg University

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*Published in:*  
Resource and Energy Economics

*Publication date:*  
1994

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*  
Withagen, C. A. A. M. (1994). Pollution and exhaustibility of fossil fuels resource. *Resource and Energy Economics*, 16(3), 235-242.

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ELSEVIER

Resource and Energy Economics 16 (1994) 235–242

RESOURCE  
and ENERGY  
ECONOMICS

## Pollution and exhaustibility of fossil fuels

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### Abstract

The use of fossil fuels causes environmental damage. This is modeled and the 'optimal' rate of depletion is derived. Also this trajectory is compared with the case where there occurs no environmental damage.

*Key words:* Pollution; Fossil fuels

*JEL Classification:* Q32; Q20

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### 1. Introduction

In the past two decades the neoclassical optimal economic growth literature, dealing with the question how to allocate resources over time so as to optimally satisfy a given objective, was faced with the need for extension in order to cope with emerging real world problems. First it was recognized that exhaustible natural resources could pose a burden to the economy's growth potential. This resulted in studies by amongst others Dasgupta and Heal (1974) and Stiglitz (1974), where the exhaustibility of natural resources was shown to play an important role. The second direction has its offspring in the field of environmental economics. The

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\* The author wishes to thank Charles van Marrewijk for useful comments. All errors remain the author's sole responsibility.

central idea is that production has pollution as an inevitable by-product, which, as a flow and/or as a stock, has a negative impact on the economy's aggregate welfare. See Brock (1977), Becker (1982), Tahvonen and Kuuluvainen (1991) and Van der Ploeg and Withagen (1991), who all add the damage caused by pollution to the social welfare function and analyse the differences with the standard Ramsey (1928) model outcomes.

Nowadays, it is widely understood that a non-negligible part of pollution is brought about by burning fossil fuels, which causes the greenhouse effect. It is not unambiguously clear whether this effect is to be considered as overall negative (in view of the fact that some agricultural areas might benefit from higher temperatures). But there is apparently a case for studying pollution and exhaustibility simultaneously. Or, as d'Arge and Kogiku (1973) state: "...the 'pure' mining problem must be coupled with the 'pure' pollution problem and questions like these become relevant: which should we run out first, air to breathe or fossil fuels to pollute the air we breathe"? We would like to add: should the stocks of fossil fuels be exhausted and what is the optimal consumption trajectory? There is some literature that deals with these questions on a rather high level of aggregation. Two approaches can be distinguished, according to where pollution affects the economy: on the production side or directly in its preferences.

Heal (1984) and Gottinger (1992) take the former line of modeling. Heal emphasizes uncertainties in the interaction between the economy and the climate. He considers a neoclassical model with raw material from an exhaustible resource as an input in the production of consumer goods and investments. The stochastic element in the model is the instant of time after which, due to cumulative resource extraction, the environment starts hindering production. This uncertainty results in "a lower rate of depletion than in the case of no climatic side effects" (Heal, 1984, p. 165). More recently, Gottinger (1992) also studies the possibly negative impact of pollution on the economy's production side, in a deterministic setting. However, exhaustibility is neglected.

D'Arge and Kogiku (1973) argue that pollution directly affects preferences. They address the question mentioned above in a model where the horizon is endogenous. It is assumed that the optimal horizon is finite. They conclude that with zero discounting it is optimal to "postpone high consumption rates until they are needed to compensate for the increasing disutility associated with rising waste density and depletion of the environment" (p. 72). In that case there is an increasing extraction rate until doomsday. A model similar to the one of d'Arge and Kogiku is analysed by Forster (1980). Forster employs a fixed horizon and allows for antipollution activities requiring the raw material (i.e. fossil fuel). He finds that antipollution activities are zero or maximal and that it may be optimal to have increasing fuel use. Here also Conrad and Clark (1987) should be mentioned, who develop an international trade model where one has to decide on allocating labour to extraction or production of the consumer commodity. Unfortunately the analysis yields only tentative results. Krautkraemer's (1985) work focuses on the question

of exhaustion. His model with resource amenities can be given an interpretation so that it fits into the type of question we wish to address. In his model the remaining stock of a natural resource enters the instantaneous utility function in a positive way, together with consumption, which is produced with the aid of the extracted commodity. There is no problem in restating the model so as to have the cumulative extraction from the resource in the instantaneous utility function, in a negative way. However, Krautkraemer only aims at finding conditions for the preservation of the natural environment (or for no exhaustion of the natural resource), meaning a positive limit as time goes to infinity. He derives such conditions for the case where technical progress in the conversion from raw material to consumer commodity exceeds society's rate of pure time preference. In the absence of technical progress exhaustion will take place. The second part of the paper by Krautkraemer incorporates physical capital into the model, as a means of production, together with the raw material, of the consumer commodity. The technology is assumed to be of the CES type. The analysis is directed again to the question of exhaustion. Finally, there is some related work by Ulph and Ulph (1992) to which we shall return in due course.

One is tempted to conclude from this overview that only minor attention is paid to the characterization of optimal exploitation patterns for the infinite horizon case. The question arises why so little has been done in this area. In view of exhaustibility it cannot be optimal to have increasing exploitation patterns as found by d'Arge and Kogiku (1973) and Forster (1980). To my opinion the reason must be mainly a technical one. In the literature on growth with exhaustible resources one is more or less forced to work with 'nice' functional forms in order to make the models tractable. Dasgupta and Heal (1974) as well as Krautkraemer (1985) employ a C.E.S. specification of the aggregate production function and a Bernoulli-type instantaneous utility function; the latter is also done by Stiglitz (1974) who uses a Cobb–Douglas production function. The problem is that in the presence of exhaustible resources one cannot hope in general for steady state solutions, but with some specific functional forms ratios of certain variables might converge. This being so it becomes quite difficult to analyse a two state– variable control problem in a more general setting. So, even when the accumulation of physical capital is disregarded as a possibility, a model with pollution and exhaustion is difficult to handle, because it necessarily entails two state variables. In this note an attempt is made to deal with both issues. The analysis will be kept as simple as possible. Nevertheless, the basic issues can be clarified in this way. We are particularly interested in the question whether or not the natural exhaustible resource will be depleted and, if the answer is in the affirmative, how the depletion trajectory looks like, compared with the trajectory where there is no social damage from pollution. At first sight it could be possible to have more consumption from the exhaustible resource initially if the stock of pollution is small. It will however turn out that with rather plausible assumptions on the functions involved this will not be the case.

## 2. The model

We consider the following problem. Maximize

$$\int_0^{\infty} e^{-\rho t} [U(c(t)) - D(s(t))] dt \quad (2.1)$$

subject to

$$\dot{x}(t) = -c(t), x(0) = x_0 \text{ given, } x(t) \geq 0 \text{ all } t \geq 0 \quad (2.2)$$

$$\dot{s}(t) = \alpha c(t) - \sigma s(t), s(0) = s_0 \text{ given.} \quad (2.3)$$

Here  $x$  is the size of the exhaustible resource and  $s$  denotes the stock of pollutants. It is assumed that aggregate consumption ( $c$ ) directly originates from the exhaustible resource (or alternatively that there is a linear conversion technology). The stock of pollutants degrades at a constant rate  $\sigma$  and the increase in pollution is proportional to consumption/production. Instantaneous social welfare is separable in consumption and pollution.  $U$  is a strictly increasing and strictly concave utility function with  $U'(0) = \infty$ .  $D$  denotes instantaneous damage.  $D$  is strictly increasing, strictly convex and satisfies  $D'(0) = 0$ . It can be doubted whether a constant rate of degradation is an adequate description of the degradation process actually going on. For the time being we shall stick to this assumption. A similar model was studied by Ulph and Ulph (1992), but they don't analyse the general case and they address the question how an optimum can be implemented in a decentralized economy, without characterizing the optimum in detail. The main formal difference with the first part Krautkraemer's paper is that he assumes  $\sigma$  to be zero, whereby from a control theoretic point of view the mathematical problem reduces to one with a single state variable.

The model is an optimal control problem with two state variables  $x$  and  $s$ . The Hamiltonian reads

$$H(c, x, s, \tilde{\lambda}, \tilde{\mu}, t) = e^{-\rho t} [U(c) - D(s)] + \tilde{\lambda}[-c] + \tilde{\mu}[\alpha c - \sigma s].$$

Assuming an interior solution for  $c$ , we have as necessary conditions

$$e^{-\rho t} U'(c) = \tilde{\lambda} - \alpha \tilde{\mu}$$

$$-\dot{\tilde{\lambda}} = 0$$

$$-\dot{\tilde{\mu}} = -e^{-\rho t} D'(s) - \sigma \tilde{\mu}.$$

Define  $\lambda := \tilde{\lambda} e^{\rho t}$ ,  $\mu := -\tilde{\mu} e^{\rho t}$ . Then the necessary conditions can be rewritten as

$$U'(c) = \lambda + \alpha \mu \quad (2.4)$$

$$\dot{\lambda} = \rho \lambda \quad (2.5)$$

$$\dot{\mu} = -D'(s) + (\rho + \sigma) \mu. \quad (2.6)$$

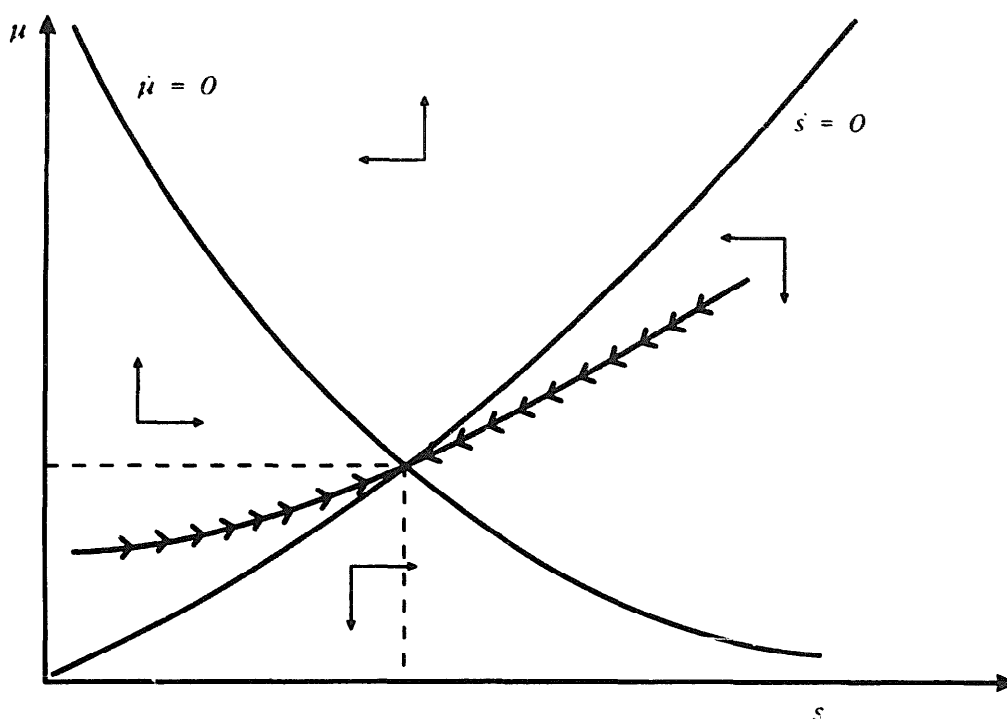


Fig. 1.

Here  $\lambda$  can be interpreted as the value attached to having an additional marginal initial unit of the exhaustible resource, whereas  $\mu(t)$  denotes the value at  $t$  of having one unit less pollution at instant of time  $t$ .

The first question we address concerns the exhaustion of the resource. One could argue that with high disutility of pollution it would be beneficial for the economy not to exhaust the resource. It is easily shown that in the present model this conjecture is incorrect, due to the assumption that marginal utility at zero consumption is infinity. The argument can be given in a simple diagram. See Fig. 1. Suppose there exists  $\bar{x} \geq 0$  such that  $x(t) > \bar{x}$  for all  $t$ . Then, obviously,  $\lambda = 0$ . Hence we have the following system.

$$U'(c) = \alpha\mu$$

$$\dot{\mu} = -D'(s) + (\rho + \sigma)\mu.$$

In Fig. 1 the loci  $\dot{s} = 0$  and  $\dot{\mu} = 0$  are depicted.

If  $\sigma > 0$  these loci have a unique point of intersection, to which the optimal solution necessarily converges (otherwise  $\mu$  becomes zero within finite time, which is not allowed, or  $s$  goes to zero but such a trajectory is overtaken by the path converging to the steady state). However a positive stock of pollution and thereby a constant rate of consumption cannot be maintained in view of the exhaustibility of the resource. Therefore  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$ . For  $\sigma = 0$  exhaustion will also take place. This can be seen as follows. If there would not be exhaustion, the stock of pollutants would be bounded and therefore also the marginal damage

caused by pollutants. Consumption however goes necessarily to zero so that marginal utility goes to infinity. But since the marginal damage of pollutants is bounded this cannot be an optimal strategy.

Now define  $\bar{\lambda}$  as the co-state variable of the resource stock in the case  $D(s) \equiv 0$ . Then we have

$$U'(\bar{c}) = \bar{\lambda} e^{\rho t}, \quad \int_0^{\infty} \bar{c} dt = x_0. \quad (2.7)$$

In the present model we have

$$U'(c) = \lambda e^{\rho t} + \alpha \mu(t), \quad \int_0^{\infty} c dt = x_0. \quad (2.8)$$

We wish to compare  $c$  and  $\bar{c}$ . Two special cases are easy to analyse. If  $\sigma = \infty$ , then basically there is no pollution problem:  $c$  and  $\bar{c}$  will coincide. If  $\sigma = 0$ , then our optimal control problem can be reduced to a one-state problem, because  $s(t) = s_0 + x_0 - x(t)$ . The necessary conditions are then

$$\begin{aligned} U'(c) &= \varphi \\ \dot{\varphi} &= \rho\varphi - \alpha D'(s). \end{aligned}$$

Hence

$$U'(c) = \varphi(t) = -e^{\rho t} \int_0^t \alpha e^{-\rho \tau} D'(\tau) d\tau + e^{\rho t} \varphi(0). \quad (2.9)$$

Now  $c(0) < \bar{c}(0)$  because otherwise  $c(t) > \bar{c}(t)$  for all  $t$  (see 2.7 – 2.9) which is ruled out. Therefore, in the case of pollution damage with  $\sigma = 0$  there is initially less consumption than without pollution damage.

These preliminary results suggest that for all  $\infty > \sigma \geq 0$  there will initially be less consumption. This is correct as can be seen as follows. Consider Fig. 2.

Suppose that  $c(0) > \bar{c}(0)$ . Since total consumption must be equal along the trajectories there exists  $t_1$  such that  $t_1$  is the first point of intersection of the two trajectories. And there exist  $t_0 < t_1 < t_2$ , such that

$$\int_{t_0}^{t_2} [c(t) - \bar{c}(t)] dt = 0.$$

It is evident that the utility of consumption on the  $\bar{c}$  trajectory is larger than on the supposed optimal trajectory:

$$\int_{t_0}^{t_2} e^{-\rho t} [U(\bar{c}) - U(c)] dt > 0.$$

Now consider the disutility of pollution. If the economy would follow the  $\bar{c}$  trajectory from  $t_0$  on, there would be less pollution than along the supposed optimal trajectory, at least up to  $t_1$ . But also for some time after  $t_1$  there would be less pollution, say, without loss of generality, up to  $t_2$ . But this being so we have

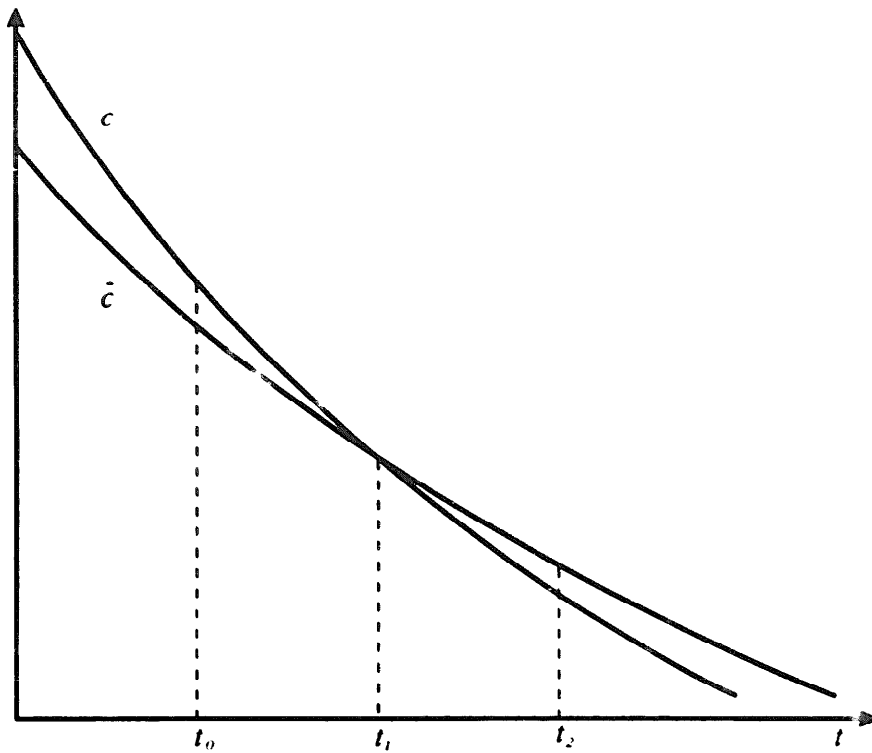


Fig. 2.

obtained a contradiction, since now it is preferable to follow  $\bar{c}$  instead of  $c$ . Therefore  $c(0) < \bar{c}(0)$  if  $\sigma < \infty$ .

It is well-known that transversality conditions in infinite horizon problems are sufficient rather than necessary. Benveniste and Scheinkmann (1982) have identified a class of control problems for which a necessary transversality condition takes an easy form. It can be shown that our problem falls into that class. Therefore, a necessary condition for optimality of a program is that

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu(t) = 0.$$

It is also easily seen that

$$\lim_{t \rightarrow \infty} \frac{U'(c(t))}{U'(\bar{c}(t))} = \lim_{t \rightarrow \infty} \frac{\lambda_0 + \alpha \mu(t) e^{-\rho t}}{\bar{\lambda}_0} = \frac{\lambda_0}{\bar{\lambda}_0}.$$

It follows that there exists  $T$  such that  $c(t) > \bar{c}(t)$  for all  $t > T$ .

The optimal trajectory can now be described as follows. Initially there is less consumption than in the pure mining model. Later there will be more consumption.

It is fairly easy to perform a sensitivity analysis. Higher emission/output rates will decrease initial consumption, higher rates of time preference will have the opposite effect.



### 3. Conclusions

In the present paper a modest attempt is undertaken to study optimal extraction of an exhaustible resource in the presence of negative externalities arising from the consumption of the raw material. It was found that the economy should initially consume (produce) less than in the case of no externalities. What makes this result interesting is that it holds irrespective of the initial stock of pollution. As soon as pollution is perceived as damaging or potentially damaging a decrease in consumption is in order. Admittedly, the model which yields this result is rather peculiar, and it should be extended into several directions, so as to allow for substitutes, physical capital, abatement activities etc. Nevertheless we hope that this analysis offers a starting point for more research in this area.

### References

- d'Arge, R. and K. Kogiku, 1973, Economic growth and the environment, *Review of Economic Studies* 40, 61–77.
- Becker, R., 1982, Intergenerational equity: the capital-environment trade-off, *Journal of Environmental Economics and Management* 9, 165–185.
- Benveniste, L. and J. Scheinkman, 1982, Duality theory for dynamic optimization models of economics, *Journal of Economic Theory* 27, 1–19.
- Brock, W., 1977, A polluted golden age, in: V. Smith, ed., *Economics of natural and environmental resources* (Gordon and Breach).
- Conrad, J. and C. Clark, 1987, *Natural resource economics* (Cambridge University Press, Cambridge).
- Dasgupta, P. and G. Heal, 1974, The optimal depletion of exhaustible resources, *Review of Economic Studies*, Symposium Issue, 3–28.
- Forster, B., 1980, Optimal energy use in a polluted environment, *Journal of Environmental Economics and Management* 7, 321–333.
- Gottinger, H., 1991, Optimal energy use under global environmental constraints, in: R. Pe'hig, ed., *Conflicts and cooperation in managing environmental resources* (Springer Verlag, Berlin) 134–181.
- Heal, J., 1984, Interaction between economy and climate: a framework for policy design under uncertainty, in: V. Smith and A. White, eds., *Advances in applied microeconomics* (JAI Press) 151–168.
- Krautkraemer, J., 1985, Optimal growth, resource amenities and the preservation of natural environments, *Review of Economic Studies* 51, 153–170.
- Van der Ploeg, F. and C. Withagen, 1991, Pollution control and the Ramsey problem, *Environmental and Resource Economics* 1, 215–236.
- Stiglitz, J., 1974, Growth with exhaustible resources: Efficient and optimal growth paths, *Review of Economic Studies*, Symposium Issue, 139–152.
- Tahvonen O. and J. Kuuluvainen, 1991, Optimal growth and stock pollution, in: F. Dietz, F. van der Ploeg and J. van der Straaten, eds., *Environmental policy and the economy* (North-Holland, Amsterdam) 47–60.
- Ulph, A. and D. Ulph, 1992, The optimal time path of a carbon tax, mimeo (University of Southampton).